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Impact of Thermal in Stokes Second Problem for Unsteady Second Grade Fluid Flow

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Abstract- This paper tosses light on the impact of thermal in Stokes second problem for unsteady second grade liquid move through porous medium. The expressions for the temperature field and the velocity field are acquired scientifically. The impacts of different rising parameters on the velocity field and the temperature field are considered through graphs in detail.

Keywords: Unsteady Second grade fluid, Porous medium, Thermal.

1 INTRODUCTION

Recently, the study of non- Newtonian fluids drew considerable attention due to their pragmatic applications. As the non- Newtonian fluids and its applications are being vital in modern technology and industries, research on such fluids are imminent. A great number of technologically and industrially vital fluids such as polymers, molten plastics, fossil fuels, foods and, pulps which may douse in underground bedsteads, display non-Newtonian behavior. Many non-Newtonian fluid prototypes have been proposed, owing to density of fluids and their properties. Therefore this category of non-Newtonian fluids, second grade fluid is the modest subclass for an analytic solution can be practically possible to discover. If the non- Newtonian fluids correspond to physically realistic situations, meticulous analytic solutions for the flows of such fluids are most likely to find, as they serve a dual purpose. Firstly, they deliver a resolution to the flow that has technical bearing. Secondly, the solutions aforementioned can be used as authorizations against intricate arithmetical cods that have been brought up for much more complex flows. Non-Newtonian fluids were studied under different Physical aspects in the recent past by Hayat et al., [2], Fetecau and Fetecau[1], Chen et al., [3], Tan and Masuoka, [5], Fetecau and Fetecau, [4].

Chauhan and Olkha [6] ventured to study the impact of space temperature dependent hear source/sink whrn heat radiation over porous stretched sheet was present. Different models of the second grade liquid issue have been contemplated by Hayat et al. [8], Baris and Dokuz [7],. Khan et al. [10], Makanda et al. [12], Hameed et al. [9] and Akinbobola [11] are contemplated magnetic and heat transfer in a vertical tube on the peristaltic transport of a second grade liquid.

The impact of temperature subordinate viscosity on viscoelastic liquid, for example, second grade liquid causes changes in the properties of the liquid. For gases, the viscosity increments as temperature increments while for fluid it diminishes as temperature increments. Thus, a lot of research work has been committed to think about the impacts of many variable consistency models.
Massoudi and Phuoc [13] utilized Reynolds Viscosity model to research the impact of variable viscosity in a completely developed flow of non-Newtonian fluid down a heated inclined plane. A similar Reynolds law was utilized in summed up second gradefluid between two vertical parallel dividers by Massoudi et al. [14]. Ramya et al [15] Studied the impacts of temperature dependent viscosity on flow and heat transfer in a viscoelastic liquid in a permeable medium. They accepted that the viscosity shifts conversely as a component of temperature. Different unidirectional transient flows of a second grade liquid in a space with one limited measurement are considered by R Bandelli et al [16].

The development of a viscoud liquid caused by the sinusoidal faltering of a level plate is named as Stokes' second issue by Schliching [17]. At first, both the plate and fluid are thought to be very still. At time t = 0+, the plate all of a sudden begins oscillating with the velocity $U_0 e^{i\omega t}$. The investigation of the flow of a viscous fluid over a swaying plate isn't just of principal hypothetical premium yet it likewise happens in many connected issues, for example, acoustic spilling around a oscillating body, an unsteady boundary layer with changes and so on (Tokuda,[18]). Penton [19] has introduced a closed form to the transient component of the solution for the stream of a viscous fluid because of a oscillating plate. Puri and Kythe [20] have talked about an unsteady flow issue which manages non-classical heat conduction impacts and the structure of waves in Stokes' second issue. Erdogan [21] broke down the unsteady flow of viscous fluid because of a oscillating plane divider by utilizing Laplace transform technique. Vajravelu and Rivera [22] talked about the hydro magnetic flow at a oscillating plate. Much work has been distributed on the flow of fluid over a oscillating plate for various constitutive models Erdogan, [23], Asghar et al., Puri and Kythe,[20], [25], Zeng andWeinbaum, [24], Ibrahem et al., [26].

In perspective of these, we thought about thermal effect in Stokes second issue for shaky second grade liquid move through permeable medium. The articulations for the temperature field and the velocity field are gotten scientifically. The impacts of different developing parameters on temperature field and the velocity field are considered through graphs in detail.

2 MATHEMATICAL FORMULATION

We consider the one-dimensional unsteady flow of a laminar, incompressible second grade liquid through a permeable medium past a vertical level plate in the yz - plane and occupy the space $x > 0$, with x-axis in the vertical direction. The plate initially at rest and at constant temperature θ_∞ which is the free stream temperature is moved with a velocity $U_0 e^{i\omega t}$ in its own plane along the z-axis, and its temperature is subjected to a periodic heating of the form (θ_w- θ_∞)$e^{i\omega t}$, where $\theta_w \neq \theta_\infty$ is some constant.

Viscoelastic fluids can be modeled by Rivlin – Ericksen constitutive equation

$$S = -p\mathrm{I} + \mu \mathrm{A}_1 + \alpha_1 \mathrm{A}_2 + \alpha_2 \mathrm{A}_1^2 \tag{2.1}$$

where p is the scalar pressure, S is the Cauchy stress tensor, μ, α_1 and α_2 are the material constants, usually known as the coefficients of viscosity, elasticity and cross - viscosity, respectively. These material constants can be resolved from viscometric flows for any real fluid. A_1 and A_2 are Rivlin-Ericksen tensors and they denote, respectively, the rate of strain and acceleration. A_1 and A_2 are defined by

$$\mathrm{A}_1 = \nabla \mathrm{V} + (\nabla \mathrm{V})^T \tag{2.2}$$

and $$\mathrm{A}_2 = \frac{d\mathrm{A}_1}{dt} + \mathrm{A}_1(\nabla \mathrm{V}) + (\nabla \mathrm{V})^T \mathrm{A}_1 \tag{2.3}$$

The viscoelastic liquids when modeled by Rivlin-Ericksen constitutive condition are termed as second-grade liquids. An detailed account of the characteristics of second - grade liquids is all around archived by Dunn and Rajagopal [28]. Rajagopal and Gupta [29] have considered the thermodynamics as dissipative inequality (Clausius – Duhem) and regularly acknowledged the particular Helmholtz free energy should be a minimum in equilibrium. From the thermodynamics thought they accepted

$$\mu \geq 0, \qquad \alpha_1 > 0, \qquad \alpha_1 + \alpha_2 = 0 \tag{2.4}$$

We seek the velocity field of the form

$$\left(u(x,t), 0, 0\right) \tag{2.5}$$

For this type of flow, equation of continuity is identically satisfied and the balance of linear momentum reduces to the following differential equation (Fetecau and Fetecau [4])

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2} + \alpha_1 \frac{\partial^3 u}{\partial x^2 \partial t} - \frac{\mu}{k} u + \rho g \beta (\theta - \theta_0) \quad \ldots (2.6)$$

The energy equation (MCF model) is given by (Ibrahem et al., 2006)

$$\tau \theta_{tt} + \theta_t = \frac{\chi}{\rho c_p} \theta_{xx} \tag{2.7}$$

Introducing the following non dimensional variables

$$\bar{x} = \frac{u_0}{\nu} x, \quad \bar{u} = \frac{u}{u_0}, \quad \bar{t} = \frac{{u_0}^2}{\nu} t, \quad \bar{\theta} = \frac{\theta - \theta_0}{\theta_w - \theta_0}$$

into the Eqs. (3.2.6) and (3.2.7), we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial^3 u}{\partial x^2 \partial t} + G\theta - \frac{1}{Da} u \tag{2.8}$$

$$p\lambda \frac{\partial^2 \theta}{\partial t^2} + p \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} \tag{2.9}$$

where

$$\alpha = \frac{\alpha_1 u_0^2}{\mu \nu}, \; Da = \frac{k u_0^2}{\nu^2}, G = \frac{\nu g \beta (\theta_w - \theta_0)}{U_0^3},$$

$$p = \frac{\nu \rho c_p}{\psi}, \;\; \lambda = \frac{\tau U_0^2}{\nu}.$$

The corresponding dimensions are boundary conditions are

$$u(0,t) = e^{i\omega t}, \qquad \theta(0,t) = e^{i\omega t}$$

$$u(\infty,t) = 0, \qquad \theta(\infty,t) = 0 \tag{2.10}$$

3 SOLUTION

To solve the non-linear system 2.3 and 2.4 using the boundary conditions (2.10), we assume that

$$u(x,t) = U(x)e^{i\omega t}\ ,\ \theta(x,t) = \Theta(x)e^{i\omega t} \tag{3.1}$$

Substituting Eq. (3.1) into Eqs. (2.3) and (2.4) and the boundary conditions (2.10), we get

$$\frac{d^2U}{dx^2} - m^2U = -Gne^{-kx} \tag{3.2}$$

$$\frac{d^2\Theta}{dx^2} + \left(\lambda p\omega^2 - i\omega p\right)\Theta = 0 \tag{3.3}$$

here $m^2 = \dfrac{\dfrac{1}{Da} + \omega^2\alpha + i\omega\left(1 - \dfrac{\alpha}{Da}\right)}{1+\omega^2\alpha^2}$ and $n = \dfrac{1 - i\omega\alpha}{1+\omega^2\alpha^2}$.

The boundary conditions are

$$U(0) = 1, \Theta(0) = 1$$

$$U(\infty) = 0, \Theta(\infty) = 0 \tag{3.4}$$

Solving the equations (3.2) - (3.3) using the boundary conditions Eq. (3.4), we obtain

$$\theta = e^{-kx} \tag{3.5}$$

$$U = e^{-mx} + \frac{Gn}{k^2 - m^2}\left[e^{-mx} - e^{-kx}\right] \tag{3.6}$$

where

$$k = \sqrt{-\lambda p\omega^2 + i\omega p} = \sqrt{\omega P\left(\frac{\sqrt{\omega^2\lambda^2+1} - \lambda\omega}{2}\right)} + i\sqrt{\omega P\left(\frac{\sqrt{\omega^2\lambda^2+1} + \lambda\omega}{2}\right)}$$

.

4 RESULTS AND DISCUSSION

Figs. 1 - 8 show the effects of various values of the emerging parameters α, G, p and Da on the velocity ($\mathrm{Re}\,u$ and $|u|$) profiles.

Fig. 1 shows the effects of material parameter α on $\mathrm{Re}\,u$ for $Da = 0.1$, $p = 1$, $\omega = 10, t = 0.1$, $\lambda = 0.005$ and $G = 5$. It is found that, the $\mathrm{Re}\,u$ decreases with increasing α. The same trend is observed from Fig. 2 for $|u|$.

Fig. 3 depicts the effects of G on $\mathrm{Re}\,u$ for $Da = 0.1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$. It is observed that, the $\mathrm{Re}\,u$ initially increases
and then decreases with increasing G.

Effects of G on $|u|$ for $Da = 0.1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$ is shown in Fig. 4. It is noted that, the $|u|$ increases with an increase in G.

Fig. 5 shows the effects of Da on $\mathrm{Re}\,u$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1, \lambda = 0.005$ and $\alpha = 0.01$. It is found that, the $\mathrm{Re}\,u$ first increases and then decreases with increasing Da.

Fig. 6 represents the effects of Da on $|u|$ for $G = 5, p = 1, \omega = 10, t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$. It is observed that, the $|u|$ increases with an increase in Da.

Effects of p on $\mathrm{Re}\,u$ for $G = 5$, $Da = 0.1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$ is shown in Fig. 7. It is found that, the $\mathrm{Re}\,u$ first decreases and then increasing with increasing p.

Effects of p on $|u|$ for $G = 5$, $Da = 0.1$, $\omega = 10$, $t = 0.1, \lambda = 0.005$ and $\alpha = 0.01$ is depicted in Fig. 8. It is noted that, the $|u|$ decreases on increasing p.

5 CONCLUSIONS

We studied the effect of thermal in Stokes second problem for unsteady second grade fluid flow through porous medium. The expressions for the velocity field and the temperature field are obtained analytically. It is found that, the $\mathrm{Re}\,u$ first decreases and then increases with increasing α or p, while it first decreases and then decreases with increasing G or Da. Further it is observed that, the $|u|$ decreases with increasing α or p, while it increases with increasing G or Da.

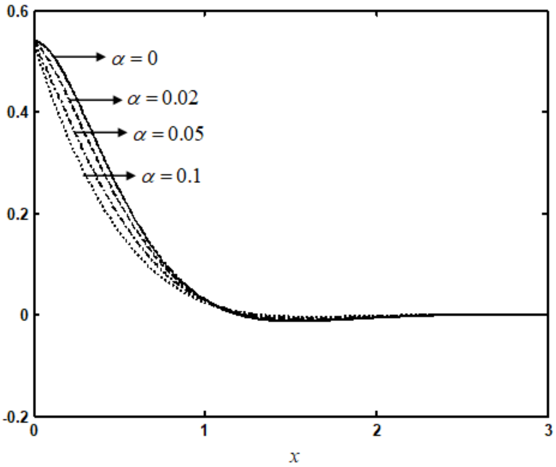

Fig. 1. Effects of α on $\mathrm{Re}\,u$ for $Da = 0.1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $G = 5$.

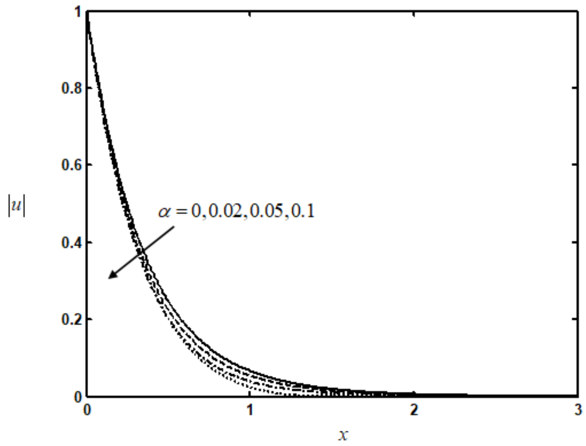

Fig. 2 Effects of α on $|u|$ for $Da = 0.1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $G = 5$.

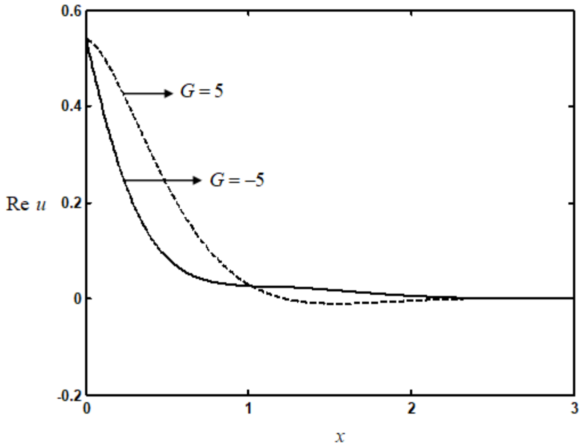

Fig. 3. Effects of G on Re u for $Da = 0.1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$.

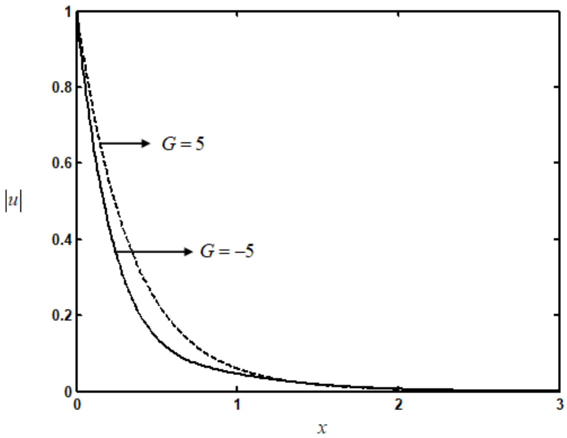

Fig. 4. Effects of G on $|u|$ for $Da = 0.1$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$.

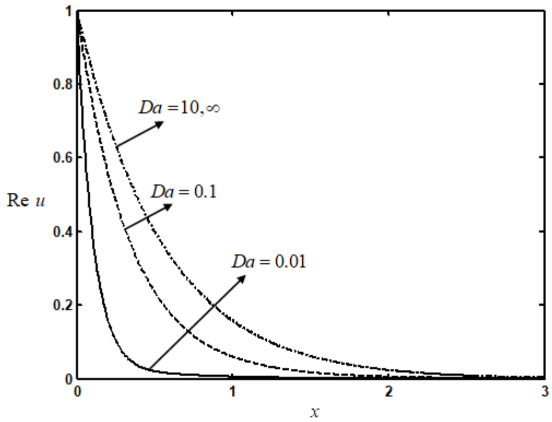

Fig. 6. Effects of Da on $|u|$ for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$.

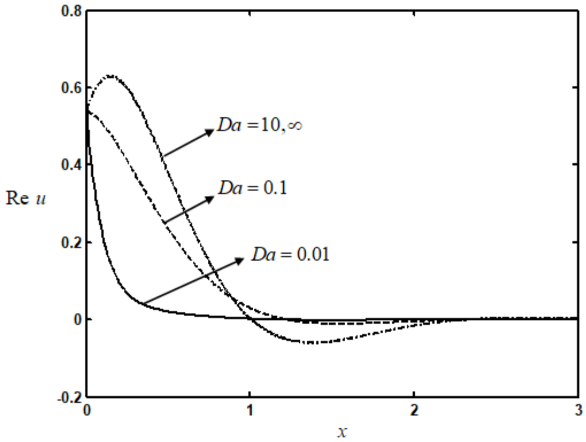

Fig. 5. Effects of Da on Reu for $G = 5$, $p = 1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$.

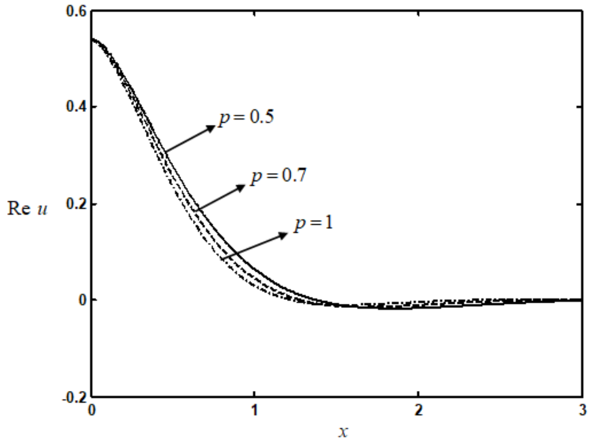

Fig. 7. Effects of p on Reu for $G = 5$, $Da = 0.1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$.

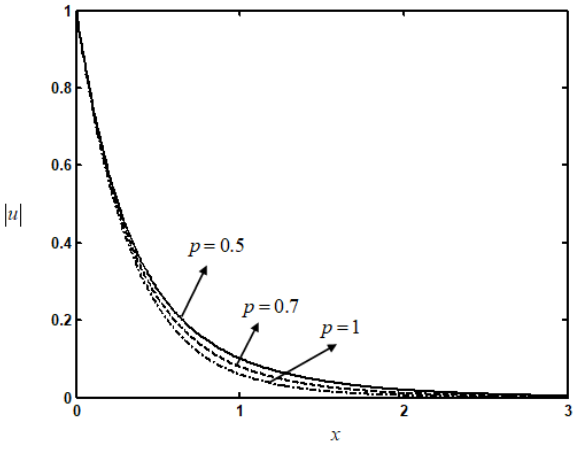

Fig. 8. Effects of p on $|u|$ for $G = 5$, $Da = 0.1$, $\omega = 10$, $t = 0.1$, $\lambda = 0.005$ and $\alpha = 0.01$.

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